Performance of OOK and Variants of PPM in APD based Free Space Optical Communication Systems

Pooja Gopal
Dept. of Electrical and Electronics Engg.
Birla Institute of Technology and Science
Pilani, Rajasthan-333031
Email: poogop@gmail.com

V. K. Jain
Dept. of Electrical Engg.
Indian Institute of Technology Delhi
Hauz Khas, New Delhi-110016
Email: vkjain@ee.iitd.ac.in

Subrat Kar
Dept. of Electrical Engg.
Indian Institute of Technology Delhi
Hauz Khas, New Delhi-110016
Email: subrat@ee.iitd.ac.in

***Abstract*—On-Off Keying (OOK) and Pulse Position Modulation (PPM) are the most commonly used modulation techniques in Free Space Optical (FSO) communications. In this paper, the performance of an FSO system with OOK and various PPM schemes has been analysed. The log-normal turbulence model for weak atmospheric turbulence and Avalanche Photo Diode (APD) receiver are considered for the system performance evaluation. The bit-error-rate (BER) performance for various schemes have been analysed and compared graphically. It was found that the performance of Differential Amplitude Pulse Position Modulation (DAPPM) is better than that of other schemes for the same peak power.**

I. Introduction

There is an increasing shift from Radio Frequency (RF) to Free Space Optical (FSO) communications because of the ever rising demand for bandwidth and also the various advantages offered by this technology. FSO system offers extremely high bandwidths, implying that it can support higher data rates. It is as yet free from the spectrum licensing problem. Also FSO systems are comparatively cheap, compact, light-weighted, easily deployable and consume lesser power. An FSO system is limited by various constraints like the requirement of precise alignment and exposure to all kinds of randomness like atmospheric turbulence because of the unguided nature. But various techniques are being proposed to mitigate the limitations of FSO and this new technology is rapidly gaining momentum.

The modulation technique used has a direct impact on the performance of a communication system and has to be chosen carefully. Power efficiency, bandwidth efficiency, simplicity of implementation and resilience to channel induced dispersion and noise are some of the important parameters against which a modulation technique is evaluated [1]. If we compare On-Off Keying (OOK) and Pulse Position Modulation (PPM), we find that PPM is more power efficient and resilient to noise than OOK. In contrast to this, OOK is simpler to implement, bandwidth efficient and resilient to channel induced dispersion than PPM. Therefore, depending on the requirement of the application, a particular modulation technique is chosen.

PPM is the most preferred modulation technique when the distances involved are very large. Hence it is used widely in satellite communications and deep space communications. Several variants of PPM are proposed, each with its own unique advantage. Performance evaluation of OOK and various PPM schemes in atmospheric turbulence is made in this paper.

The paper is organized as follows: Section II describes the modulation techniques considered. In section III the system description is given in brief. In section IV the proposed methodology for the system evaluation is given. The numerical analysis and results are given in section V. The conclusions are given in section VI.

II. Intensity Modulation Techniques

Although all the phase coherent modulation techniques developed for RF communications can also be used for FSO communications, their implementation is still difficult because of the high frequencies involved. The ability of Intensity Modulation with Direct Detection (IM/DD), which is unique to optical communications, offers more advantages. It is a well-known fact that by relying entirely on the energy of the signal (as in the case of IM/DD) and completely ignoring the coherence of the radiation does not fundamentally limit the rate at which data can be sent over a noiseless channel. Also because of the simplicity in implementation, IM/DD systems are widely used in optical communications.

PPM is an orthogonal modulation technique commonly used in optical communications because of its unparalleled power efficiency. This is because an M-ary PPM symbol has exactly one pulse and $M-1$ empty slots implying low average power and hence a high Peak to Average Power Ratio (PAPR). This makes it an ideal choice for applications in which power is critical. The required bandwidth as compared to OOK is large for higher order PPM. Hence, PPM falls back on bandwidth efficiency as compared to OOK.

In Differential Pulse Position Modulation (DPPM), all the empty slots following the pulse in a PPM symbol are removed. Hence, the average symbol length reduces implying improved bandwidth efficiency. Also, there is an inherent symbol synchronization capability because every symbol ends with a pulse. There could be a problem in slot synchronization with DPPM, when a series of zero bits i.e., a series of pulses are encountered. To overcome this, the use of one or more guard band(s)/slot(s) immediately after the pulse has been proposed [2].

In Dual Header Pulse Interval Modulation (DHPIM) [3], each symbol can have one of the two predefined headers H_0 and H_1 depending on the input information. If the most significant bit (MSB) of the input code is 0 then H_0 is chosen. Likewise, H_1 is chosen if the MSB is 1. The header is followed by d empty slots. The value of d is simply the decimal value of the M-1 least significant bits of input data when H_0 is the header and the 1's complement of M-1 bits when H_1 is the header. Since this reduces the average symbol length compared to DPPM, bandwidth efficiency is improved.

Differential Amplitude Pulse Position Modulation (DAPPM) [4] has advantages over the other variants of PPM in terms of the bandwidth requirement at the cost of increased power requirement and complexity of the decoding circuit. The DAPPM is a combination of DPPM and Pulse Amplitude Modulation (PAM). The length of a DAPPM symbol varies from 1 to L and the pulse amplitude level is selected from 1 to A, where A and L are integers. A is optimally 2^k ($k \in I$), but not necessarily so. The average number of empty slots in a symbol can be reduced by increasing the number of amplitude levels A and thereby increasing the achievable throughput.

Table I shows the symbol mapping of all the variants of PPM considered with OOK for a bit resolution of 4. One guard slot is considered for DHPIM and DAPPM to avoid synchronization problem when a series of zero bits (pulses) are received.

OOK	PPM	DPPM	DHPIM	DAPPM
0000	1000000000000000	1	100	01
0001	0100000000000000	01	1000	001
0010	0010000000000000	001	10000	0001
0011	0001000000000000	0001	100000	00001
0100	0000100000000000	00001	1000000	02
0101	0000010000000000	000001	10000000	002
0110	0000001000000000	0000001	100000000	0002
0111	0000000100000000	00000001	1000000000	00002
1000	0000000010000000	000000001	1100000000	03
1001	0000000001000000	0000000001	110000000	003
1010	0000000000100000	00000000001	11000000	0003
1011	0000000000010000	000000000001	1100000	00003
1100	0000000000001000	0000000000001	110000	04
1101	0000000000000100	00000000000001	11000	004
1110	0000000000000010	000000000000001	1100	0004
1111	0000000000000001	0000000000000001	110	00004

TABLE I. SYMBOL MAPPING OF PPM, DPPM, DHPIM AND DAPPM (A=4) FOR A BIT RESOLUTION OF m=4

III. SYSTEM DESCRIPTION

The three most reported models for irradiance fluctuations in a turbulent channel are: log-normal, gamma-gamma and negative exponential. Their respective ranges of validity, are in the weak, weak-to-strong and saturation regimes.

The irradiance fluctuations in the received beam is measured in terms of scintillation index σ_I^2, which is defined as [5]

$$\sigma_I^2 = \frac{< I^2 > - < I >^2}{< I^2 >} \tag{1}$$

where I is the irradiance at a point in the receiver plane and $< >$ denotes an ensemble average.

The intensity statistics for weak turbulence condition which corresponds to $\sigma_I^2 < 1$ is given by the log-normal distribution. $\sigma_I^2 \geq 1$ corresponds to strong turbulence conditions for which the field amplitude is Rayleigh distributed. This means negative exponential statistics for the intensity distribution. The gamma-gamma distribution is used to describe the intensity statistics for the entire range of turbulence. Although it is mathematically more complex, the gamma-gamma distribution can relate to various values of scintillation index unlike the other distributions which are valid only for limited range of values.

The effect of atmospheric turbulence on a ground to satellite FSO link is much less than that of a terrestrial FSO link and the scintillation index σ_I^2 is experimentally found to lie between 0.28 and 1.12 [6]. Hence the log-normal turbulence model is quite appropriate for analysing the effect of turbulence on a ground to satellite FSO link.

The probability density function of log-normal distribution is given by [7]

$$p(I) = \frac{1}{\sqrt{2\pi\sigma_I^2}}\frac{1}{I}exp\left\{-\frac{(ln(I/I_0) - E[I])^2}{2\sigma_I^2}\right\}, \quad I \geq 0 \tag{2}$$

where I is the received field intensity in the presence of turbulence and I_0 the received field intensity without the effect of turbulence, σ_I^2 the log-intensity variance and $E[I]$ the mean of log-intensity variance.

The Avalanche Photo Diode (APD) is generally used in long distance optical communications because of the low received power levels. An APD performs better than a PIN diode, when the received power levels are low. A high avalanche gain requires a high reverse bias voltage. The higher gain does not imply a better signal to noise ratio since the performance degrades beyond a certain gain as the effect of noise becomes dominant. Hence the optimum gain of APD for a particular system has to be used.

IV. PROPOSED METHODOLOGY FOR SYSTEM EVALUATION

With an optimum decision threshold, the BER depends only on the Q-parameter and is given by [8]

$$BER = \frac{1}{2}erfc\left(\frac{Q}{\sqrt{2}}\right) \tag{3}$$

where

$$Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0} \tag{4}$$

I_1 and I_0 are the photo-detector currents corresponding to the detection of '1' and '0' bit, respectively and σ_0^2 and σ_1^2 are the corresponding noise variances.

In terms of the number of received photons, the APD photo-detector currents corresponding to '1' and '0' bits can be written as

$$I_1 = \frac{qN_sg}{T} \tag{5a}$$

and

$$I_0 = 0 \tag{5b}$$

where q is the electronic charge, N_s the number of received signal photons, T the bit duration and g the average gain of the APD.

To calculate the noise variance, we assume that the APD has a negligibly small dark current. So, only shot noise, thermal noise and background noise are considered. The shot noise variance is given by [8]

$$\sigma_s^2 = 2qg^2FI_pB \tag{6}$$

I_p is either I_1 or I_0, depending on the received bit. B is the receiver bandwidth taken to be $1/T$. Since I_0 is considered zero, the shot noise associated with a '0' bit is zero. In the above equation, F is the noise figure of the APD, and is given by [8]

$$F = \zeta g + (2 - 1/g)(1 - \zeta) \tag{7}$$

where ζ is the ionization constant. The thermal noise is given by [8]

$$\sigma_T^2 = \frac{4kT_0}{R_L} B \tag{8}$$

where *k* is the Boltzmann constant, T_0 the receiver temperature in Kelvin and R_L the load resistance. The noise variance due to background radiation is given by,

$$\sigma_B^2 = 2qg^2 F I_B B \tag{9}$$

where I_B is the background noise current. In terms of the number of background photons N_B, it can be written as

$$I_B = \frac{qN_B g}{T} \tag{10}$$

The noise variances when '1' bit and '0' bit are received, are respectively given by

$$\sigma_1 = \sqrt{\sigma_s^2 + \sigma_T^2 + \sigma_B^2} \tag{11}$$

$$\sigma_0 = \sqrt{\sigma_T^2 + \sigma_B^2} \tag{12}$$

When the free space optical signal passes through weak turbulence channel, the number of received photons N_s at the APD receiver is a log-normal distributed random variable. Its pdf from eqn. (2) is given by

$$p(N_s) = \frac{1}{\sqrt{2\pi\sigma_y^2} N_s} exp\left[-\frac{(ln(N_s) - m_y)^2}{2\sigma_y^2}\right], \quad N_s \geq 0 \tag{13}$$

where m_y and σ_y^2 are the mean and variance of $ln(N_s)$, respectively.

The scintillation index σ_I^2 is related to σ_y^2 by

$$\sigma_y^2 = ln(\sigma_I^2 + 1) \tag{14}$$

Also, the mean m_y is related to σ_y^2 and the average number of signal photons received $E[N_s]$ by

$$m_y = ln(E[N_s]) - \sigma_y^2/2 \tag{15}$$

A. *OOK Modulation*

From eqs. (4) and (5), we see that the Q-parameter is a function of the number of received signal photons N_s, which is a log-normal distributed random variable. Hence, the BER is conditioned on the random variable N_s. From eqn. (3), the conditional BER is given by

$$P_{b/i} = \frac{1}{2} \, erfc\left[\frac{Q(N_s)}{\sqrt{2}}\right] \tag{16}$$

The unconditional BER is then given by

$$BER = P_b = \int_0^\infty P_{b/i} \, p(N_s) dN_s \tag{17}$$

After statistical averaging over the turbulent channel and further simplification by using the Gauss-Hermite approximation [9], we get

$$BER = P_b = \frac{1}{2\sqrt{\pi}} \sum_{i=1}^{N} w_i \, erfc\left[\frac{Q(x_i)}{\sqrt{2}}\right] \tag{18}$$

where w_i and x_i are the weights and zeros of the Hermite polynomial [10].

B. *M-ary Pulse Position Modulation (PPM)*

The computation of the slot error rate for the case of M-ary PPM is similar to the computation of *BER* of OOK with the exception of change in certain parameters. First of all, m bits of OOK correspond to 2^m=M bits of PPM. To achieve the same throughput, the data rate must be increased by a factor of M/m. Also, the signal and background photons detected at the receiver during one slot duration, is reduced by the same factor. An M-ary PPM symbol is decoded incorrectly if one or more slots in the symbol are in error. Hence, the Symbol Error Rate (SER) is of more relevance than BER. We consider that each slot being in error or not being in error is an independent event. The probability that none of the slots are in error is obtained by multiplying the individual probabilities and is given by

$$P_{NoSymbolError} = (1 - P_b)^M \tag{19}$$

Therefore, the probability that at least one slot is in error is

$$SER = 1 - P_{NoSymbolError} = 1 - (1 - P_b)^M \tag{20}$$

C. *Differential Pulse Position Modulation (DPPM)*

The length of a DPPM symbol varies from 1 to 2^m. If we consider all the symbols to be equiprobable, then the average length of a DPPM symbol is given by

$$L_{avg,DPPM} = \frac{2^m + 1}{2} \tag{21}$$

The analysis for DPPM and the rest of the variants is similar to that of M-ary PPM, with M replaced by L_{avg}. The bit-rate as compared to OOK increases by a factor $L_{avg,DPPM}/m$ and the number of signal and background photons detected per DPPM slot is reduced by the same factor.

Differential Pulse Interval Modulation (DPIM) is similar to DPPM, where the number of pulse intervals following the pulse determine the symbol transmitted. A DPIM symbol is nothing but a mirror image of the corresponding DPPM symbol.

D. *Differential Pulse Interval Modulation (DPIM)*

DPIM symbols are nothing but the mirror image of DPPM symbols. Hence the analysis and results are the same for both. Therefore, it is not considered here.

E. Dual Header Pulse Interval Modulation (DHPIM)

Due to the presence of the header bits, the maximum number of pulse intervals in a symbol, as compared to DPIM is reduced by a factor of 2. If α represents the number of header bits, the length of a DHPIM symbol with one guard bit varies from 1+α to $2^{m-1}+\alpha$. Considering all symbols to be equiprobable, the average length of a DHPIM symbol is given by

$$L_{avg,DHPIM} = \frac{2^{m-1}+2\alpha+1}{2} \quad (22)$$

As compared to OOK, the data rate increases by a factor of $L_{avg,DHPIM}/m$ and the number of signal and background photons reduce by the same factor. In the numerical computations reported in Section 5, the value of α is chosen to be 2.

F. Differential Amplitude Pulse Position Modulation (DAPPM)

Unlike the case of a two level modulation scheme, in the case of DAPPM an error can occur as a bit being read as any one of the remaining A-1 levels. But practically, an error most likely occurs when a level is read as one of the adjacent levels. Certain modifications to the general BER formula given in eqn. (16) are required for this case.

There are A-1 decision levels, which the incoming bits are compared with. Here we make an assumption that the noise variances at all levels are equal and that decision levels are midway between the two adjacent signal levels. Therefore, eqn. (4) can be extended as

$$Q = \frac{I_1 - I_0}{\sigma_1+\sigma_0} = \frac{2I_1 - I_1}{\sigma_2+\sigma_1} = \frac{3I_1 - 2I_1}{\sigma_3+\sigma_1} = \ldots \quad (23)$$

The equality in the above equation holds because I_0=0 and also we have assumed that $\sigma_0 = \sigma_i$ $(i = 1, 2, 3\ldots)$.

Except the received bits of level '0' and level 'A', all other bits can be misread as being either of the adjacent levels. Level '0' and level 'A' can only be misread as level '1' and level 'A-1', respectively. Therefore, we can write the equation for BER as

$$BER = p(0)P(1|0) + p(1)[P(0|1) + P(2|1)] + \ldots$$

$$\ldots + p(A)P(A-1|A) \quad (24)$$

The probability of a level being misread is given by

$$P(1|0) = \ldots = P(A-1|A) = \frac{1}{2}erfc\left(\frac{Q}{\sqrt{2}}\right) \quad (25)$$

If A levels are chosen, then the 2^m symbols are divided into A groups, the symbols in each group having 0 to $2^m/A$ zeros (including 1 guard bit) followed by a pulse having one of the A amplitude levels. To avoid synchronization problem, a single pulse interval is used as a guard band. The DAPPM symbol length varies from 1 to $2^m/A$. The average number of zeros in a symbol is then $(2^m+A)/2A$. Considering equiprobable symbols, the average symbol length is given by

$$L_{avg,DAPPM} = \frac{2^m+3A}{2A} \quad (26)$$

The probability of occurrence of a '0' bit is

$$p(0) = \frac{2^m+A}{2^m+3A} \quad (27)$$

The probability of occurrence of any other level is

$$p(1) = p(2) = \ldots = p(A) = 1 - p(0) = \frac{2A}{2^m+3A} \quad (28)$$

Therefore from eqs. (23) to (25),

$$BER = p(0)\frac{1}{2}erfc\left(\frac{Q}{\sqrt{2}}\right) + p(1)erfc\left(\frac{Q}{\sqrt{2}}\right) + \ldots +$$
$$\ldots + p(A-1)erfc\left(\frac{Q}{\sqrt{2}}\right) + p(A)\frac{1}{2}erfc\left(\frac{Q}{\sqrt{2}}\right) \quad (29)$$

From eqs. (27) to (29), we get

$$BER = \left(\frac{2^m+4A^2-A}{2(2^m+3A)}\right)erfc\left(\frac{Q}{\sqrt{2}}\right) \quad (30)$$

After statistical averaging over the turbulent channel and further simplification by using the Gauss-Hermite approximation [9], similar to the OOK case, we get

$$P_b = \frac{1}{2\sqrt{\pi}}\left(\frac{2^m+4A^2-A}{2^m+3A}\right)\sum_{i=1}^{N} w_i\, erfc\left[\frac{Q(x_i)}{\sqrt{2}}\right] \quad (31)$$

As in the earlier cases, in this case bit rate is increased by a factor of $L_{avg,MDPIM}/m$ and the number of background photons will be reduced by the same factor, as compared to OOK.

Parameter	Value
Bit rate ($1/T$)	1 Gbps
Receiver temperature (T_0)	300 K
Average signal photon count ($E[N_s]$)	400
Background photon count (N_b)	20
Average APD gain (g)	200
Load resistance (R_L)	100 Ω
Ionization constant (ζ)	0.028
APD noise figure (F)	6.756

TABLE II. PARAMETER VALUES IN NUMERICAL COMPUTATION

V. Numerical Analysis and Results

For the case of weak atmospheric turbulence, the value of the scintillation index σ_I^2 must be less than 1. The key system parameters used in computation are given in Table II.

Variations of SER with APD gain for OOK and various PPM schemes for turbulence with scintillation index (σ_I^2) of 0.2 is shown in Fig. 1. The corresponding graphs for turbulence with σ_I^2 of 0.7 is given in Fig. 2. It is observed that the optimum APD gain is found to be approximately 200 for all the schemes and different turbulent conditions.

In Fig. 3, the plots of *SER* vs. the number of received signal photons for the various schemes is plotted for turbulence with scintillation index (σ_I^2) of 0.2, considering the same peak power in all cases. It is observed that the performance of the DAPPM scheme is better than that of the rest. The corresponding graph for turbulence with σ_I^2 of 0.7 is given in Fig. 4.

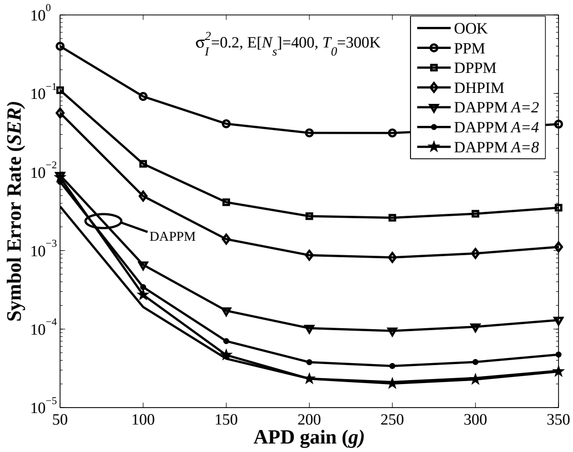

Fig. 1. SER vs. g for OOK and various PPM schemes at σ_I^2=0.2

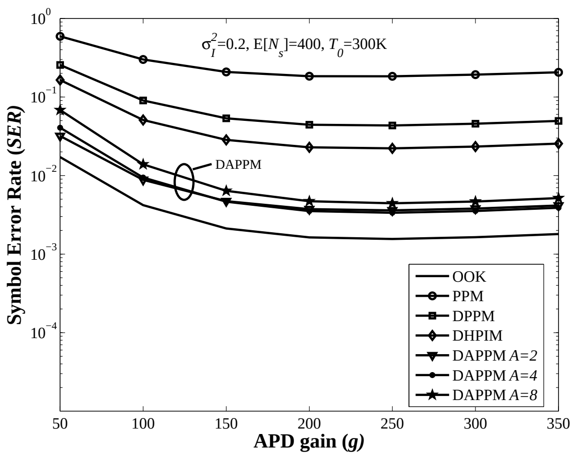

Fig. 2. SER vs. g for OOK and various PPM schemes at σ_I^2=0.7

VI. Conclusion

In this paper, BER expressions for OOK and various PPM schemes for APD receiver and log-normal atmospheric turbulence have been derived.

It is observed that the performance of DAPPM is better than that of the rest. It is also observed that by increasing the number of levels A, the performance degrades. This is due to the fact that more number of levels will give rise to higher probability of bit error.

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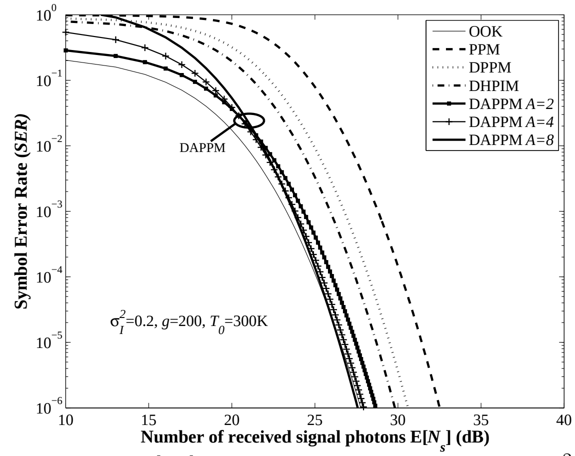

Fig. 3. SER vs. $E[N_s]$ for OOK and various PPM schemes at σ_I^2=0.2

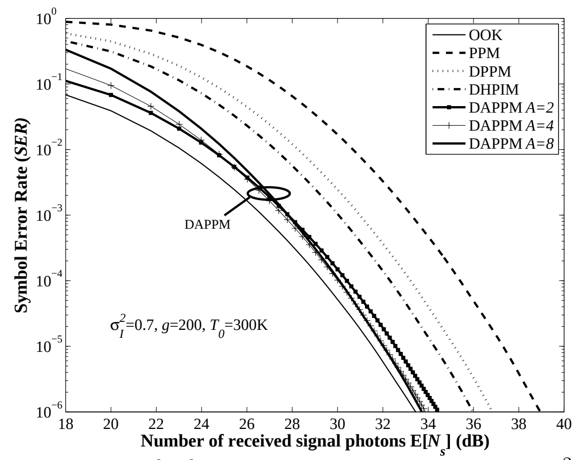

Fig. 4. SER vs. $E[N_s]$ for OOK and various PPM schemes at σ_I^2=0.7

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